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SPACES OF MULTI-ANISOTROPIC ULTRADIFFERENTIABLE FUNCTIONS

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1. ABSTRACT

Let $\mathscr{A}(\Omega)$ denotes the classical space of real analytic functions defined on a non empty open set Ω of the Euclidean space $\mathbb{R}^n$. Different classes of ultradifferentiable functions have been considered and studied as an extension or a generalisation of the space $\mathscr{A}(\Omega)$. The following scheme illustrates some ones of this development :

$$\begin{array}{ccc} & & \mathscr{E}^{\omega}(\Omega) \\ & \nearrow & \\ \mathscr{A}(\Omega) & \to & \mathscr{E}^{M}(\Omega) \\ & \searrow & \\ & & \mathscr{E}^{s,\Gamma}(\Omega) \end{array}$$

The ultradifferentiable function classes are denoted as follows

1. $\mathscr{E}^{M}(\Omega)$ is the Denjoy-Carleman space, see [14],[15],[20].
2. $\mathscr{E}^{\omega}(\Omega)$ is the Beurling space, see [1], [**?**], [13].
3. $\mathscr{E}^{s,\Gamma}(\Omega)$ is the multi-anisotropic Gevrey space, see [16], [22].

Remark 1 *It is not necessary to remind the importance of these spaces in mathematics, and particularly in partial differential equations, [19], [17] and [21].*

A recent paper [2] addressed the issue to find the relationship between the spaces $\mathscr{E}^{M}(\Omega)$ and $\mathscr{E}^{\omega}(\Omega)$.

So, firstly this work deals with the question of finding a relationship between the Denjoy-Carleman space $\mathscr{E}^{M}(\Omega)$ and the multi-anisotropic Gevrey space $\mathscr{E}^{s,\Gamma}(\Omega)$, and then it introduces the space of multi-anisotropic ultradifferentiable functions $\mathscr{E}^{M,\Gamma}(\Omega)$ unifying these both spaces.

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