# NUMERICAL SIMULATION OF A FINITE ELEMENT BENDING SUPPORT

*Abdelkader kirad*[1]*,Loukmane Zeddam*[2]

[12]Department of Mechanical Engineering, Faculty of Technology
[12]Fundamental and Applied Physics FUNDAPL
[12]University of Blida 1, BP 270 Blida 09000, Algeria.

**ABSTRACT**

The work presented consists on the one hand of researching the values of the stresses and displacements of a plate subjected to a perpendicular external force, the distribution of the stress is determined at the end of the localization of the maximum stress. On the other hand, verified that the natural frequency is the excitation frequency, so as not to have a resonance. The equilibrium equations are obtained by applying the principle of virtual work. The mathematical expressions of displacements, normal and tangential stresses are obtained by using the Navier approach to solve the equilibrium equation system. The stiffness matrix and mass matrix are calculated by exact integrations based on the finite element method. The results obtained agree well with those given in the bibliography and those obtained by using the ANSYS apdl code for the calculation of finite elements. Key words : finite elements, stress, displacement, force, a plate, frequency, resonance.

### 0.1. List of symbols

$\{\varepsilon\}$ : deformation vector.
$\{\sigma\}$ : constraint vector.
$\{q\}$ : displacement vector.
[K] : rigidity matrix.
v : Poisson coefficient.
$\beta$ : represents the beat angle [rad]
$\xi$ : represents the angle of drag [rad]
$\theta$ : represents the angle of torsion [rad]
$f$ : The frequency expressed in Hertz [Hz]
$A$ : The maximum amplitude [m]
T : The kinetic energy of the system [Joule]
U : The potential energy of the system [Joule]
$\{u\}^e$ : Vector of displacements at a point of the element.
[u(x,y,z)] : Matrix of the base functions of the spatial approximation
[N(x,y,z)] : Matrix of interpolation functions for the element.
$\{q(t)\}$ : Vector of displacements at the nodes of the element.
[M] : The mass matrix.
F(t) : The external force applied to the structure.

# 1. INTRODUCTION

The calculation of complex structures requires the implementation of increasingly sophisticated mechanical behavior modeling tools, taking into account the specifications of these material-structures. From a practical point of view, numerical methods, in particular calculation by the finite element method, are essential for the design and verification of complex structures. The Finite Element Method (FEM) is a numerical analysis technique allowing approximate solutions to a wide variety of engineering sciences. The basic principle of FEM is to replace the real domain with a set of discrete elements, which can be brought together in several ways, and therefore use them to represent the most complex shapes.. MEF with the advent of computers first became in 1956 a general approximation formulation of structural mechanics when used at Boeing by Turner, Clough, Martin and Top, to calculate parts of the structure of an airplane, this work marked the birth of the Finite Element method.

The applications of the finite element method are divided according to the nature of the problem to be solved into three categories. In the first category, we find the problem of equilibrium which is part of. the field of solid mechanics, where one needs to know the displacements, the strains and the stresses for a given mechanical or thermal loading, of which one finds the majority of MEF applications. In the second category, we find the eigenvalue problems, these are stationary problems whose solution often requires the determination of the natural frequencies and the modes of vibration of solids and fluids The control of vibrations in structural elements such as plates is a thorny problem that frequently arises for the engineer, to ensure this control, the determination of the dynamic characteristics of the plates is essential. The chosen mathematical model is reliable if the required response is known to be predicted within a chosen level of accuracy measured on the response of the full mathematical model

# 2. DEFINITION AND GENERAL NOTATIONS :

## 2.1. bending stiffness matrix based on Kirchhoff theory :

To simplify the study, consider a rectangular element with four nodes of dimension (a, b, e) with this time three (03) D.D.L

For each node, (W, $\theta_x$et$\theta_y$)as this element has 12 D.D.L, we will have the same member of generalized coordinates. From this effect, we can approximate the displacements to :

$$(x,y) = d_1 + d_2x + d_3y + d_4x^2 + d_5xy + d_6y^2 + d_7x^3 + d_8x^2y + d_9xy^2 + d_{10}y^3 + d_{11}x^3y + d_{12}xy^3 \tag{1}$$

We make the following change of variable :

$$\zeta = \frac{x}{a} \quad ; \quad \eta = \frac{y}{b} \tag{2}$$

Then the function w (x, y) becomes :

$$W(\zeta,\eta) = c_1 + c_2\zeta + c_3\eta + c_4\zeta^2 + c_5\zeta\eta + c_6\eta^2 + c_7\zeta^3 + c_8\zeta^2\eta + c_9\zeta\eta^2 + c_{10}\eta^3 + c_{11}\zeta^3\eta + c_{12}\zeta\eta^3 \tag{3}$$

The displacement vector {q} is written :

$$\{q\}_F^t = \{w_1, \theta_{x1}, \theta_{y1}, w_2, \theta_{x2}, \theta_{y2}, w_3, \theta_{x3}, \theta_{y3}, w_4, \theta_{x4}, \theta_{y4}\} \tag{4}$$

On the other hand, on each node the corresponding load system consists of the two (02) moments $M_x, M_y$ and 1 a shear force $P_z$ then, the force vector is written

$$\{P\}^t = \{P_{z1}, M_{x1}, M_{y1}, P_{z2}, M_{x2}, M_{y2}, P_{z3}, M_{x3}, M_{y3}, P_{z4}, M_{x4}, M_{y4}\} \tag{5}$$

Les forces nodules et les déplacements sont relies par :

$$\{P\} = [K]\{q\} \tag{6}$$

To find the constants $C_i(i = 1,2,3,\ldots,12)$ ) it is necessary to set the boundary conditions.
for the node 1 : $(\zeta = 0, \eta = 0) \rightarrow (x = 0, y = 0)$
for the node 2 : $(\zeta = 1, \eta = 0) \rightarrow (x = a, y = 0)$
for the node 3 : $(\zeta = 1, \eta = 1) \rightarrow (x = a, y = b)$
for the node 4 : $(\zeta = 0, \eta = 1) \rightarrow (x = 0, y = b)$
Alor :

$$\{\delta(\zeta\eta)\}_F = [g(\zeta\eta)]_F[H]_F^{-1}\{q\}_F \tag{7}$$

### 2.2. Relation deformation displacement :

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2} \tag{8}$$

$$\varepsilon_y = -z\frac{\partial^2 w}{\partial y^2} \tag{9}$$

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z(\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial y}) = -2z\frac{\partial^2 w}{\partial x\partial y} \tag{10}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = -z\begin{Bmatrix} \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial y^2} \\ \frac{\partial^2}{2\partial x\partial y} \end{Bmatrix} w \tag{11}$$

Taking into account the change of variable we will have :

$$\frac{\partial^2}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2}{\partial \zeta^2} \tag{12}$$

$$\frac{\partial^2}{\partial y^2} = \frac{1}{a^2}\frac{\partial^2}{\partial \zeta^2} \tag{13}$$

$$\frac{\partial^2}{\partial x\partial y} = \frac{1}{ab}\frac{\partial^2}{\partial \zeta a\eta} \tag{14}$$

Then the deformation expression becomes :

$$\begin{Bmatrix} \varepsilon_\zeta \\ \varepsilon_\eta \\ \gamma_{\zeta\eta} \end{Bmatrix} = -Z\begin{Bmatrix} \frac{\partial^2}{a^2\partial \zeta^2} \\ \frac{\partial^2}{b^2\partial \eta^2} \\ \frac{\partial^2}{ab\partial\zeta\partial\eta} \end{Bmatrix} W \tag{15}$$

By calculating each term we obtain :

$$\varepsilon_\zeta = -Z\frac{1}{a^2}(2c_4 + 6c_7\zeta + 2c_8\eta + 6c_{11}\zeta\eta) \quad \varepsilon_\eta = -Z\frac{1}{b^2}(2c_6 + 6c_9\zeta + 6c_{10}\eta + 6c_{12}\zeta\eta) \tag{16}$$

Replacing the expression in the deformation formula we get :
In matrix form we have :

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_\zeta \\ \varepsilon_\eta \\ \gamma_{\zeta\eta} \end{Bmatrix} - Z[\varnothing(\zeta\eta)]_F\{c\}_F \tag{17}$$

Such as :

$$[\varnothing(\zeta\eta)]_F = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{a^2} & 0 & \frac{6\zeta}{a^2} & \frac{2\eta}{a^2} & 0 & \frac{6\zeta\eta}{a^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\zeta}{b^2} & 0 & \frac{6\zeta\eta}{b^2} \\ 0 & 0 & 0 & 0 & \frac{2}{ab} & 0 & \frac{4\eta}{ab} & \frac{4\eta}{ab} & \frac{6\zeta^2}{ab} & \frac{6\eta^2}{ab} \end{pmatrix} \tag{18}$$

Replacing the expression $\{c\}_F = \{H\}_F^{-1}\{q\}$ in the deformation formula we get :

$$\{\varepsilon\} = -Z[\varnothing(\zeta\eta)]_F\{H\}_F^{-1}\{q\}_F \tag{19}$$

We ask :

$$[B(\zeta\eta)]_F = [\varnothing(\zeta\eta)]_F\{H\}_F^{-1}[\varepsilon(\zeta\eta)] = -Z[B(\zeta\eta)]\{q\}_F \tag{20}$$

### 2.3. Relation stresses deformations :

$$\{\sigma(\zeta\eta)\} = [E]\{\varepsilon(\zeta\eta)\} \tag{21}$$

Substituting the equation of the expression $\{\varepsilon\}$ we will have

$$\{\sigma(\zeta\eta)\} = -Z[E][B(\zeta\eta)]\{q\} \tag{22}$$

### 2.4. Determination of the bending stiffness matrix :

The calculation of the stiffness matrix is done by the following formula :

$$[K]_F = \int v[B]^t[E][B]dv \tag{23}$$

The 78 values of the matrix [K] are calculated using the following formula :

$$K_{ij} = \frac{abh^3}{12}\int_0^1\int_0^1\{(B_{1i}B_{1j})e_{11} + (B_{2i}B_{1j} + B_{1i}B_{2j})e_{12} + (B_{2i}B_{2j})e_{22} + (B_{3i}B_{3j})e_{33}\}d\zeta d\eta \tag{24}$$

For
$i = 1,2,3,\ldots,12$
$i = i, i+1,\ldots,12$
To calculate the elements $K_i j$ we can use a program that calculates the numerical integral of the functions

## 3. DYNAMIC STUDY :

### 3.1. Elementary formulation :

The formulation at the element level consists in expressing the kinetic and potential energies as a function, respectively, of the speeds and displacements at the nodes, that is :

$$V^e = U^e - W^e = \frac{1}{2}\{q^e\}^T[K^e]\{q^e\} - q^{e^T}\{F^e\} \tag{25}$$

With : $[K^e] = \int_{V^e}[B]^T[E][B]dV$

$$\{F^e\} = \int_{V^e}[N]^T\{F\}dV + \int_{S^e}[N]^T\{P\}dS \tag{26}$$

We remind you that we have :

$$\{\delta\} = [E][B]\{q\} \tag{27}$$

Regarding the kinetic energy of the element we have :

$$T^e = \frac{1}{2}\int_{V^e}\rho\{\dot{u}\}^{e^T}\{\dot{u}\}^e dV \tag{28}$$

And like :

$$\{u\}^e = [N]\{q\} \tag{29}$$

Then :

$$\{\dot{u}\}^e = [N]\{\dot{q}\} \tag{30}$$

$$T^e = \frac{1}{2}\{\dot{q}\}^{e^T}[M]^e\{\dot{q}\}^{e^T} \tag{31}$$

$[M]^e = \int_V \rho [N]^T [N] dV$ :Matrix Mass of the element
The calculation is carried out by numerical integration and only involves the density, the geometry and the interpolation matrix of the element.

### 3.2. Global formulation :

The Lagrange equations starting from obtaining the discrete equations of motion, either for a structure without damping :

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = F_i, \quad i = 1,2,3\ldots N \tag{32}$$

We have :

$$T = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\} \tag{33}$$

$$U = \frac{1}{2}\{q\}^T[K]\{q\} \tag{34}$$

From where we get :

$$[M]\{\ddot{q}\} + [K]\{q\} = F(t) \tag{35}$$

The displacement vector {q} is written :

$$\{q\}^T = \{u_i v_j, u_j v_j, ukv_k\} \tag{36}$$

### 3.3. Rectangular element of membrane with quadratic fields :

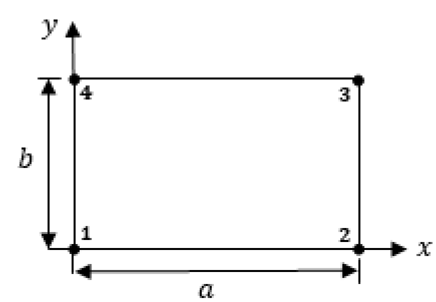


FIGURE 1 – Elément rectangulaire à huit degrés de liberté

$$[M] = \rho e \int_0^a \int_0^b [N]^T[N] dxdy = e\rho ab \int_0^1 \int_0^1 [N(\xi,\eta)]^T[N(\xi,\eta)] d\xi d\eta \tag{37}$$

Then : after substitution of these functions in the expression of [M] we get :

$$[M] = \rho eab \begin{bmatrix} \frac{1}{9} & & & & & & & \\ 0 & \frac{1}{9} & & & s & y & m & \\ \frac{1}{18} & 0 & \frac{1}{9} & & & & & \\ 0 & \frac{1}{18} & 0 & \frac{1}{9} & & & & \\ \frac{1}{36} & 0 & \frac{1}{18} & 0 & \frac{1}{9} & & & \\ 0 & \frac{1}{36} & 0 & \frac{1}{18} & 0 & \frac{1}{9} & & \\ \frac{1}{18} & 0 & \frac{1}{36} & 0 & \frac{1}{18} & 0 & \frac{1}{9} & \\ 0 & \frac{1}{18} & 0 & \frac{1}{36} & 0 & \frac{1}{18} & 0 & \frac{1}{9} \end{bmatrix} \tag{38}$$

## 4. PREPROCESSOR :

In the preprocessor the model is installed, it includes a number of steps ; usually in the following order : - Buildgeometry : construction of the geometry, according to the geometry of the problem one, two dimensions or three-dimensional. - Definematerials : a material is defined by its material constants. - Generateelementmesh (Mesh generation) : The problem is discretized with the nodes.

### 4.1. Processor solution :

When you solve the problem by gathering all the specific information about the problem : - Apply loads : Boundary conditions are generally applied to nodes or elements. - Optain solution : The solution of the problem can be obtained if the whole problem is defined.

### 4.2. Post processor :

In this part of the analysis you can for example : - Visualise the results (Visualiser les résultats) - Liste the results (Listes des résultats)

### 4.3. Plaque étudié :

Materials : $S275 \quad e = 5mm \quad F_1 = F_2 = 50N \quad q_1 = 0.4N \quad q_2 = 1N \quad r_1 = 12.5mm$ $r_2 = r_3 = 2.5mm$ ; the plate has a Young's modulus of E=200 GPa , and a Poisson's ratio v=0.3 ; and density = 7.85 10-6kg.mm3 ; Re=275 MPa

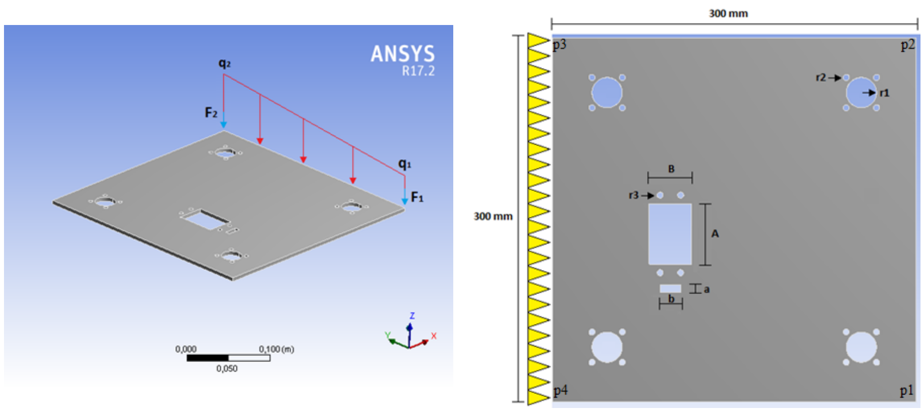


FIGURE 2 – Support studied

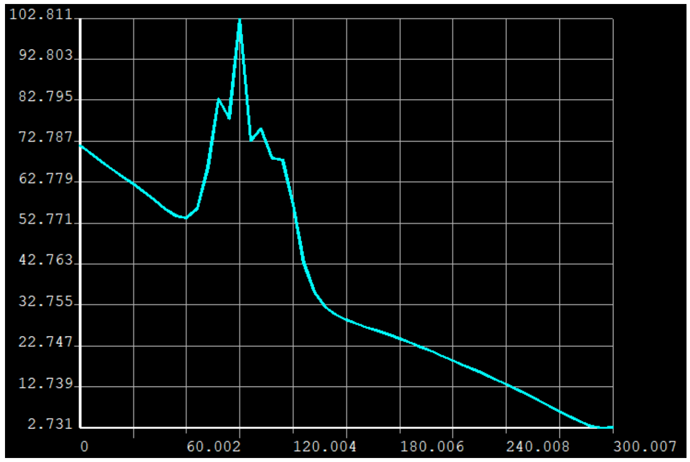


FIGURE 3 – stress eqv

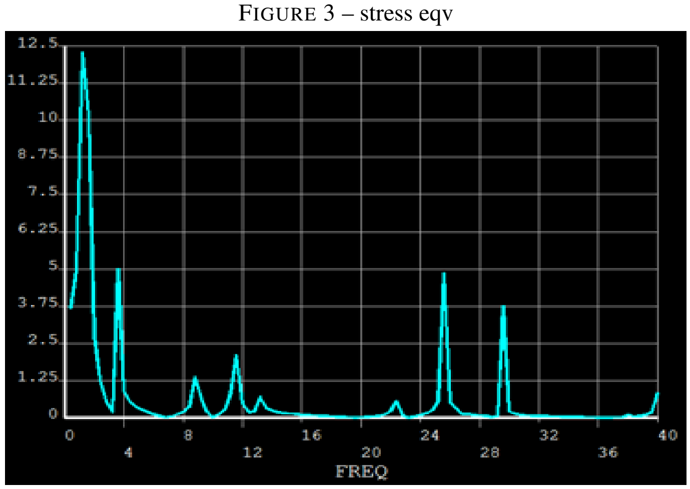


FIGURE 4 – vibration amplitude in p1

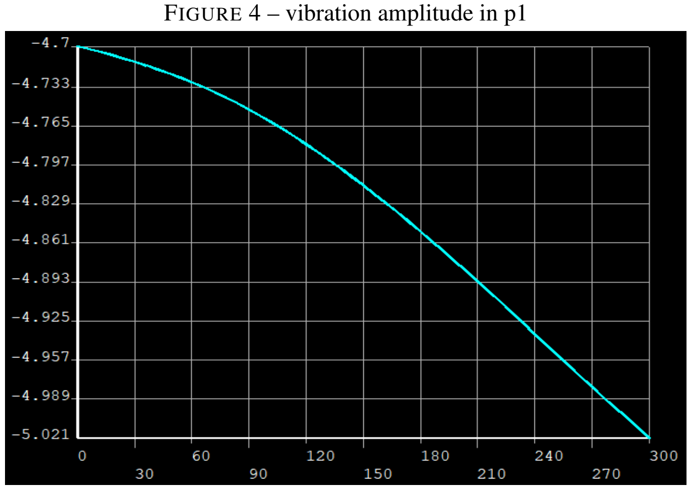


FIGURE 5 – displacement in axis p1-p2

## 5. CONCLUSIONS

The work carried out relates to the finite element method modeling of a support. The programming of the method in ANSYS allowed a more exact fundamental frequency and study the displacement, stress and the natural frequency. Thus we solve the equation of free movement to determine the natural frequencies and to avoid the phenomenon of resonance. The stress field shows that the stresses are not homogeneous in a perforated plate stressed in bending. In the vicinity of the hole, the stresses are maximum at the edge of the hole. Maximum displacement at the end where force has been applied. In study, we used structural analysis with ANSYS APDL.

## 6. THE BIBLIOGRAPHY

[1] : Batoz j L and Dhatt G. Modeling of structures by finite elements, vol.2 : beam and plate, Edition Hermès, paris 1990.

[2] : EDUQRD VENTSEL and THORDOR KRANTHAMMER. "Thin plates and shelles".

[3] : Jean-LuisBatoz, “Modeling of structures by finite elements”, volume 2, Beams and Plates.

[4] : K.J. BATHE, D. CHAPELLE, and P.S. Lee “A shell problem ’highly sensitive’ to thickness changes. Internet ". J. Number. Methods Eng., 2003

[5] : François. Frey "Analysis of structures and continuous media (Mechanics of solids)", volume3 Presses polytechniques, edition (1999).

[6] : Rahmani O and KebdaniS, Introduction to the Finite Element Method for engineer, 2nd ed. OPU, 1994.