THE M-MACHINE CHAIN-REENTRANT FLOW SHOP WITH TWO COMPETING AGENTS

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ABSTRACT

In this paper, we consider the scheduling problem on m-machine chain-reentrant flow shop problem with two competing agents. In which, all the jobs must pass through all the machines in the same predetermined order, then return back to the first machine for their last operation. The objective is to minimize the overall completion time of one agent subject to an upper bound on the makespan of the second agent. $\mathcal{NP}$-hardness proof is provided for the two-machine flow shop problem with the two competing agents with respect to the makespan criteria, even if each job has the same processing time on the machines, while a polynomial time algorithm is presented for the m-machine with unit processing time. We also develop a mathematical programming model and a heuristic for the resolution.

1. INTRODUCTION

Flow shop scheduling problems have attracted the interest of many researchers, as the number of applications for them grows rapidly. This topic entails scheduling a set of n jobs on a succession of m machines that all take the same processing route. Johnson [8] shown that the two-machine flow shop with respect to the makespan is polynomially solvable in $O(n\log n)$, whereas the three-machine problem was proven to be strongly $\mathcal{NP}$-hard in[6].

The flow shop problem asserts that each job only visits each machine once. However, due to the rising complexity of manufacturing processes, this criterion is frequently violated in practice, leading to the development of better-suited variations of the traditional flow shop. The reentrant flow shop problem is one of these versions, in which jobs are supposed to pass through the machines in the same predetermined order, with the assumption that some machines can be visited more then once, see [7]. This configuration can be encountered in several manufacturing processes such as : Automobile Assembly Line, Integrated Circuit (IC) manufacturing, Photolithography, Semiconductors, etc. For more details on this problem and its applications, one can see [5] and [14]. Many variants of reentrant shops have been introduced in the literature, the first of which is the V-shop problem, which appears for the first time in [9]. The given model has a V-shape route, which justifie the name, i.e, $(M_1, \cdots, M_m, M_{m-1}, \cdots, M_1)$. The chain-reentrant shop problem has been studied in [13] and [4], where it is assumed that each job has to be processed through all machines in the same processing order and returns back to the first machine for its last operation $(M_1, M_2, \cdots, M_m, M_1)$. Recently, [3] addressed the same issue with respect to both makespan and total tardiness performance measures. In [10], the author considered the two-stage chain-reentrant hybrid flow shop, where each stage consists of multiple identical machines and developed some lower bounds and heuristics.

Multi-agent scheduling refers to the problems in which common resources are shared by two or more agents. In most of the classical scheduling problems, it is assumed implicitly that the set of jobs are handled by a single agent, thus associated with a single criterion to be optimized. Indeed, the scarcity of resources has led decision makers to share those resources with other users, each associated with his own set of jobs, thus with his own criterion to optimize. On the other hand, the technological constraints of many real applications in various fields. For mor detail or survey, one can see [2], [11], [12] and [1].

In this paper, we consider the m-machine chain-reentrant flow shop scheduling problem with two competing agents. The objective is to minimize the makespan of one agent, while maintaining the makespan value of the second agent under a given value. The jobs are processed without preemption, from time 0, starting on M_1, then through $M2$, M_3, $\cdots$, M_m, and back to M_1. We assume that each job is processed by at most one machine at a time, and each machine can handle one job at a time. This document is structured as follows : Section 2 is dedicated to problem description and notations. Section 3 contains complexity results. Section 4 is dedicated to a mathematical model. Section 5 describe a heuristic algorithm. While Section 6 presents the results of our mathematical model and heuristic. Finally, we finish our work with a conclusion in Section 7.

2. PROBLEM DESCRIPTION AND NOTATIONS

The m-machine chain-reentrant flow shop with two competing agents studied in this paper is described as follow. Given two sets J_A and J_B of n jobs ($|J_A|+|J_B|=n$) of two competing agents A and B. Each agent X, $X \in \{A,B\}$, is associated with a set of n_X jobs $J_X = \{J_1^X, J_2^X, \cdots, J_{n_X}^X\}$ to be processed on machine M_1, goes through M_2, M_3, $\cdots$, M_m, and finally returns back to M_1 for its final execution. We point out that $J = J_A \cup J_B$, $|J_A| = n_A$ and $|J_B| = n_B$. The processing time of job J_j^X on machine M_i is denoted by $P_{i,j}^X$, $J_j^X \in J_X, X \in \{A,B\},\ i = 1,2,\cdots,m$. The goal is to find a schedule σ of the n jobs satisfying the above constraints such that the overall completion time (makespan) associated with agent A is minimized, while maintaining the makespan of agent B under a fixed value Q. If $C_j^X(\sigma)$ denotes the finish time of job J_j^X under schedule σ, then the makespan of agent X is $C_{max}^X(\sigma) = \max_{j \in J_X}\{C_j^X\}, X = \{A,B\}$. This problem is denoted $Fm|Chain-reentrant|C_{max}^A : C_{max}^B \le Q$

3. COMPLEXITY RESULTS

Theorem 1 *$F2|Chain-reentrent, p_{ij} = p_j|C^A_{max} : C^B_{max} \leq Q$ is $\mathcal{NP}$-hard even if agent A has one job.*

Theorem 2 *$Fm|Chain-reentrent, p_{ij} = 1|C^A_{max} : C^B_{max} \leq Q$ is solvable in $O(n^2)$.*

4. MATHEMATICAL MODEL

In this model, we use time-indexed variables to determine the start date operations and a finite planning horizon of length T.

4.1. Decision variables

$$x_{i,j,t} = \begin{cases} 1 & \text{If job } j \text{ start its execution at instant } t \text{ on machine } i \\ 0 & \text{Otherwise} \end{cases}$$

4.2. Objective function

$$\min y_A$$

4.3. Constraints

$$\sum_{t\in T} x_{i,j,t} = 1 \qquad \forall j \in J, i = 1, \cdots, m+1 \qquad (1)$$

$$\sum_{t\in T} (t + p_{i,j})x_{i,j,t} \leq \sum_{t\in T} t\, x_{i+1,j,t} \qquad \forall j \in J, i = 1, \cdots, m \qquad (2)$$

$$\sum_{j\in J} \sum_{t'=t-p_{i,j}+1}^{t} x_{i,j,t'} \leq 1 \qquad \forall i = 1, \cdots, m+1, t \in T \qquad (3)$$

$$(t + p_{m+1,j})x_{m+1,j,t} \leq y_A \qquad \forall t \in T, \forall j \in J_A \qquad (4)$$

$$(t + p_{m+1,j})x_{m+1,j,t} \leq Q \qquad \forall t \in T, \forall j \in J_B \qquad (5)$$

$$\sum_{j\in J} \sum_{t'=t-p_{1,j}+1}^{t} x_{1,j,t'} + \sum_{t'=t-p_{m+1,j}+1}^{t} x_{m+1,j,t'} \leq 1 \qquad t \in T \qquad (6)$$

(1) Guarantees that each job is processing exactly one time on each machine.
(2) Ensures that each job must finish its processing on M_i before starting on M_{i+1}.
(3) Ensures that at most one job is processed at each time on each machine.
(4) Verifies that J_A makespan never exceeds y_A.
(5) Prevents J_B makespan from exceeding Q.
(6) Ensures that M_1 and M_{m+1} are not processing jobs at the same time, as they are in fact one machine.

5. HEURISTIC

In this section, we present a heuristic that uses Johnson's rule to construct a solution for the tackeled problem. Denote by $JR(a^r, b^r)(\sigma)$ the solution obtained by Johnson's rule for the problem of the two machines flow shop with processing time a^r and b^r of the iteration r (a^r b^r are the processing time of the jobs on the first and the second machine, respectively).

5.1. Algorithm heuristic

Step 1 : Construction of a permutation

- Set $r = 1$;
- Set $\sigma = \phi$ and $C_{\max}(\sigma) = +\infty$;
- While $r <= m$ do
  - Set $a_j^r = \sum_{i=1}^{r} p_{ij}$ and $a_j^r = \sum_{i=r+1}^{m+1} p_{ij}$ for $j = 1, \ldots, n$;
  - Calculate the makespan of the current data using Johson rule. let σ' be the schedule obtained.
  - $C_{\max}(\sigma') = JR(a^r, b^r)(\sigma')$;
  - if $C_{\max}(\sigma') < C_{\max}(\sigma)$ then
    - Set $\sigma = \sigma'$;
    - Set $C_{\max}(\sigma) = C_{\max}(\sigma')$;
  - EndIf;
  - $i = i + 1$;
- EndWhile;

Step 2 : Construction of a solution with a single agent

- Set $\pi = (\sigma, \sigma)$, Which means each jobs is duplicated in the sequence;
- For each job in π, schedule the first one of π on $M_1, \ldots, M_m$ then the second one on M_1 only;
- Let $C_{max}(\pi)$ the total length of the schedule;

Step 3 : Constructing a solution with two agents

- Let $C_{max}(\pi)^A$ and $C_{max}(\pi)^B$ be the makespan of the agent A and B, respectively of the solution π;
- Let $\pi^1 = \pi$ and $\pi^2 = \phi$;
- Let $\delta = \pi^1 \cup \pi^2$;
- Let $C_{max}(\delta)^A$ and $C_{max}(\delta)^B$ be the makespan of the two agents;
- If $C_{max}(\pi) \leq Q$ then
  - While $C_{max}(\delta)^B \leq Q$ and an operation of agent B in π^1 do
    - Move the last operation of the agent B from π^1 to π^2;
    - Let $\delta' = \pi^1 \cup \pi^2$ and calculate $C_{max}(\delta')^A$ and $C_{max}(\delta')^B$;
    - If $C_{max}(\delta')^B \leq Q$ then
      - Set $\delta = \delta'$, $C_{max}(\delta')^A = C_{max}(\delta)^A$ and $C_{max}(\delta')^B = C_{max}(\delta)^B$;
    - Else
      - Return δ;
    - EndIf
  - EndWhile;
- Else
  - While $C_{max}(\delta)^B \leq Q$ and an operation of agent A in π^1 do
    - Move the last operation of the agent A from π^1 to π^2;
    - Let $\delta' = \pi^1 \cup \pi^2$ and calculate $C_{max}(\delta')^A$ and $C_{max}(\delta')^B$;
    - If $C_{max}(\delta')^B \leq Q$ then
      - Return δ;
    - Else
      - Set $\delta = \delta'$, $C_{max}(\delta')^A = C_{max}(\delta)^A$ and $C_{max}(\delta')^B = C_{max}(\delta)^B$;
    - EndIf
  - EndWhile;
- EndIf

6. NUMERICAL EXPERIMENTS

In this section, we present the results of our mathematical model and heuristic. The former have been executed on CPLEX (20.1), while the latter has been coded in C++ and executed on an i7-8750H CPU. 10 instances with processing times generated randomly using uniform distribution in $[1,20]$. In which, the second half of the jobs is affected to agent B, the value of Q is set to 300 for both the model and the heuristic, as well as $T = 400$ for the model. The results of the deviation are shown in Table 1 in which column C^A_{max} refer to the makespan of Agent A found by the heuristic and $C^{A^\star}_{max}$ the optimal makespan found by the mathematical model.

instance	C^A_{max}	$C^{A^\star}_{max}$	Deviation
1	99	76	30.26%
2	99	95	4.21%
3	130	126	3.17%
4	128	121	5.79%
5	87	79	10.13%
6	114	91	25.27%
7	97	93	4.30%
8	105	101	3.96%
9	152	140	8.57%
10	95	92	3.26%

TABLE 1 – Deviation of the heuristic

We can see that except for instances $1,5$ and 6 all deviations are under 10%, while 5 and 6 are still viable, instance 1 represents the worst scenario. All the instances have been solved optimally with CPLEX under 3 minutes.

7. CONCLUSIONS

In this work, we addressed the m-machine chain-reentrant flow shop with two competing agents. $\mathcal{NP}$-hardeness proof is presented for the proportionate problem, while a polynomail time algorithm is provided to solve the problem with unit processing time. A mathematical model has been proposed for the exact resolution and a heuristic for its approched resolution. In perspective, much remain to be done for the numerical experiments, more approched methodes are to be programmed and tested, as well as other complexity studies.

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