

NONLOCAL DIFFERENTIAL OPERATORS APPLIED TO IMAGE PROCESSING

*S. BEN ALIA*¹, *M. Hachama*²

¹Laboratory of AMNEDP, Departement of mathemaics,
University of Sciences and Technology Houari Boumediene.

²University of Blida 1, Algeria.

ABSTRACT

In this paper, we present a model for image shadows removal. The model reformulates a recent osmosis model with nonlocal differential operators. Experimental results show that the nonlocal model obtained very good qualitative results compared with state-of-the-art techniques.

Key Words : *Image restoration, Shadows removal, Nonlocal differential operators Energy, minimization.*

1. INTRODUCTION

We consider a given grayscale image $f : \Omega \rightarrow \mathbb{R}_+^*$, where $\Omega \subset \mathbb{R}^2$ is a regular domain with boundary $\partial\Omega$ and f is continuously differentiable. We suppose that the image f contains a shadow that we want to filter out. As shown on Figure 1, we decompose the domain Ω into three disjoint regions,

$$\Omega = \Omega_{in} \cup \Omega_{sb} \cup \Omega_{out}, \tag{1}$$

where Ω_{out} , Ω_{in} and Ω_{sb} represent the shadow-free, shadowed, and boundary regions respectively. Their characteristic functions will be denoted by χ_{out} , χ_{in} , and χ_{out} .

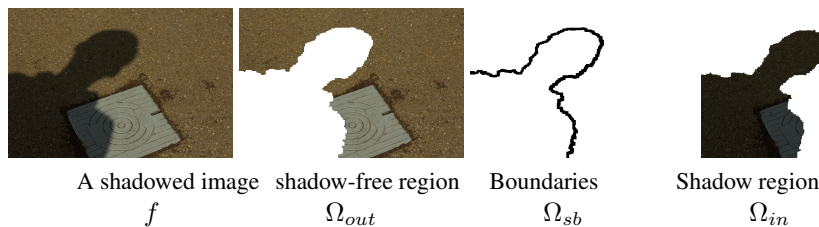


FIGURE 1 – Image shadow and domain decomposition.

2. NONLOCAL MODEL FOR IMAGE SHADOWS REMOVAL

We propose a new nonlocal model for shadow removal which consists in looking for a function $u : \Omega \rightarrow \mathbb{R}_+$ that minimizes an energy composed of three terms :

$$\mathcal{E}(u) := \mathcal{S}(u) + \lambda_1 \mathcal{R}(u) + \lambda_2 \mathcal{F}(u), \quad (2)$$

where \mathcal{S} denotes a shadow term, \mathcal{R} is a regularization term, and \mathcal{F} is a fidelity term. The terms are balanced using positive weights λ_1 and λ_2 .

Shadow term This term is the nonlocal version of the osmosis model [2] :

$$\mathcal{S}(u) = \frac{1}{2} \int_{\Omega} v(x) \left| \nabla_{NL} \left(\frac{u}{v} \right) \right|^2(x) dx, \quad (3)$$

where

$$v(x) = \begin{cases} f(x), & \text{if } x \in \Omega_{in} \cup \Omega_{out}, \\ 1, & \text{if } x \in \Omega_{sb}, \end{cases} \quad (4)$$

and $\nabla_{NL}u : \Omega \times \Omega \rightarrow \mathbb{R}$ stands for a nonlocal gradient which permits to take into account nonlocal interactions between distant pixels. Here, we use the Gilboa operator [4] defined by

$$\nabla_{NL}u(x, y) = (u(y) - u(x)) \sqrt{\omega(x, y)},$$

where $0 \leq \omega(x, y) < \infty$ is a weight function. In this paper we assume symmetric weights, i.e., $\omega(x, y) = \omega(y, x)$. The magnitude of a nonlocal gradient is defined by

$$|\nabla_{NL}u|(x) := \sqrt{\int_{\Omega} \nabla_{NL}u(x, y)^2 dy}.$$

For any $p : \Omega \times \Omega \rightarrow \mathbb{R}$, considering the usual inner products of L^2 space, the nonlocal divergence $\text{div}_{NL} p : \Omega \rightarrow \mathbb{R}$ is defined as

$$(\text{div}_{NL} p)(x) = \int_{\Omega} (p(x, y) - p(y, x)) \sqrt{w(x, y)} dy.$$

Fidelity term As the image region Ω_{out} does not contain shadows, we impose that the solution u should stay close to f on Ω_{out} . Therefore, we minimize the following term

$$\mathcal{F}(u) = \frac{1}{2} \int_{\Omega_{out}} \frac{(f(x) - u(x))^2}{f(x)} dx. \quad (5)$$

Regularization term To ensure a smooth transition between the shadowed and shadow-free regions, we perform an anisotropic regularization of image intensities on Ω_{sb} . As in [1], using the modified tensor voting approach [3], we estimate the local structure, a positive semi-definite symmetric matrix-valued field $\mathbf{W} : \Omega \rightarrow \mathbb{R}^{2 \times 2}$ used to promote preferred directions. We define the regularization term \mathcal{R} as

$$\mathcal{R}(u) = \frac{1}{2} \int_{\Omega_{sb}} \|\nabla u(x)\|_{\mathbf{W}}^2 dx, \quad (6)$$

where $\|e\|_{\mathbf{W}} := \sqrt{\langle e, \mathbf{W}e \rangle}$. This term can be interpreted as an inpainting in which information is propagated from Ω_{out} and Ω_{in} to Ω_{sb} . It is worth noting that local derivatives are considered here.

3. EVOLUTION PROBLEM

In this section, we derive the evolution associated with the energy (2). Starting from an initial image usually chosen to be f , we seek a restored image $u(t, \cdot)$ which will evolve over time t .

Proposition 1 Assume that $f \in L^\infty(\Omega; \mathbb{R}_+)$. The evolution process associated to Equation (2) is given as follows :

$$\begin{cases} \partial_t u(t, x) &= \operatorname{div}_{NL} \left(v(x) \nabla_{NL} \left(\frac{u(t, \cdot)}{v} \right) (x, y) \right) \\ &+ \lambda_1 \chi_{sb}(x) \operatorname{div}(\mathbf{W}(x) \nabla u(t, x)) \\ &+ \lambda_2 \chi_{out}(x) (f(x) - u(t, x)), & \text{in } \Omega_t, \\ u(0, x) &= f(x), & \text{in } \Omega, \\ \langle \mathbf{W} \nabla u, n \rangle &= 0, & \text{on } \partial \Omega_{sb}^t, \end{cases} \quad (7)$$

where $\Omega_t = (0, T) \times \Omega$, $\partial \Omega_{sb}^t = (0, T) \times \partial \Omega_{sb}$, and n is the outward normal.

to solve this equation, we used the explicit Euler method.

4. EXPERIMENTS

We compared our nonlocal osmosis model to previous isotropic [1] and anisotropic [2] osmosis models, on synthetic and real images. Figure 2 shows some cases. Our model improved shadow removal in most cases, overcoming blurring artifacts on the boundaries (isotropic model) and in the shadowed regions (anisotropic model). We also obtained better results in the shadow-free region.

5. CONCLUSIONS

In this paper, we presented our new nonlocal model for shadow removal. Our main contribution is the use of nonlocal derivatives within the recent osmosis model, which takes into account distant pixels similarities. Numerical resolution gives an efficient algorithm which overcomes blurring artifacts and preserves the textures and details compared to previous state-of-the-art techniques.

6. REFERENCES

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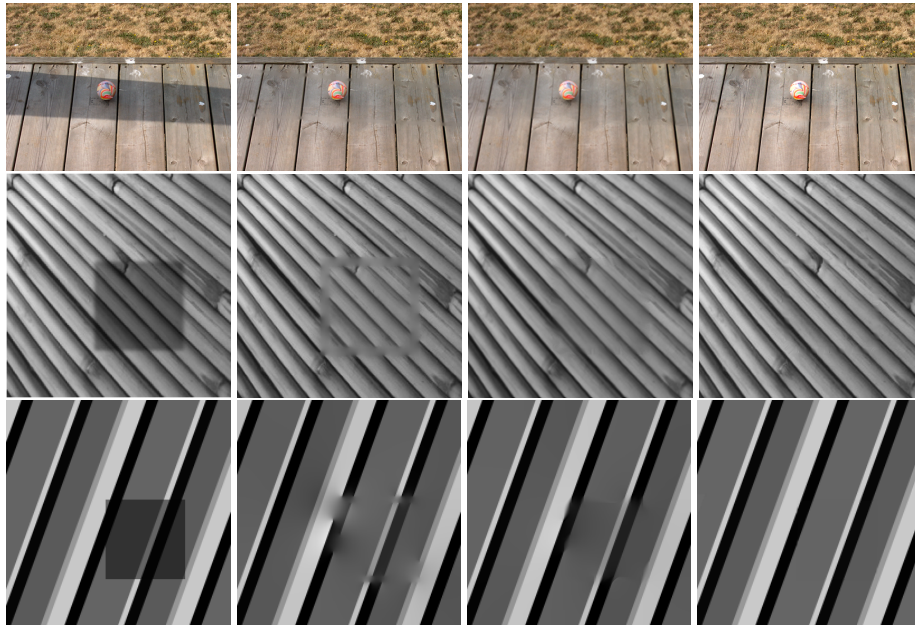


FIGURE 2 – Shadowed removal using osmosis models. From left to right : Input (shadowed) image, Isotropic [2], Anisotropic [1], and the nonlocal osmosis models results.

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