A DISCRETE FRACTIONAL COVID-19 MODEL: EXISTENCE AND STABILITY RESULTS

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ABSTRACT

In this paper we will discuss a discrete fractional order covid-19 model and give results for existence and conditions to ensure the disappearance of the disease.
KEYWORDS : Fractional difference systems - Existence - Stability - covid 19.

1. INTRODUCTION

After the spread of the Corona pandemic, many mathematicians built models to try to analyze the spread of this epidemic. Recently modeling several chemical and physical phenomena have broadly been carried out using the theory of Fractional-order Difference Systems (FoDSs). In this work, we will study the discret fractional model of one of the models of Covid 19 and the study of existence and uniqueness, and then we will present conditions to ensure the disappearance of the epidemic.

2. PRELIMINARIES

This section briefly introduces some basic definitions and preliminaries associated with discrete fractional calculus. In the whole of the definitions below, the function f is defined on the set of the form $\mathbb{N}_a = \{a, a+1, a+2, ...\}$, where $a \in \mathbb{R}$.

Definition 1 *[1] Let $\alpha > 0$. Then, the $\alpha^{th}-$fractional sum, $\Delta_a^{-\alpha}$, of a function $f : \mathbb{N}_a \to \mathbb{R}$ is defined by :*

$$\Delta_a^{-\alpha} f(t) := \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} f(s), \ \ for\ t \in \mathbb{N}_{a+\alpha}, \tag{1}$$

where $\Gamma(.)$ is the Euler's gamma function.

Definition 2 *[1] Let $\alpha > 0$, $\alpha \notin \mathbb{N}$. Then, the $\alpha^{th}-$order Caputo fractional difference of a function f is defined by :*

$${}^C\Delta_a^{\alpha} f(t) := \Delta_a^{-(n-\alpha)} \Delta^n f(t) = \frac{1}{\Gamma(n-\alpha)} \sum_{s=a}^{t-(n-\alpha)} (t-s-1)^{(n-\alpha-1)} \Delta^n f(s), \quad t \in \mathbb{N}_{a+n-\alpha}, \tag{2}$$

where $n = [\alpha] + 1$.

3. SYSTEM MODEL

Consider the model of COVID-19 suggested and studied in [2] as follow :

$$\begin{cases} \frac{dS}{dt} = & \theta - \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)S}{N(t)} - \mu S + \sigma R \\ \frac{dE}{dt} = & \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)S}{N(t)} - (\mu+\omega)E \\ \frac{dI}{dt} = & \omega E - (\mu+\delta+\rho+\tau)I \\ \frac{dQ}{dt} = & \rho I - (\mu+\delta+\phi)Q \\ \frac{dR}{dt} = & \phi Q + \tau I - (\sigma+\mu)R \end{cases} \tag{3}$$

where $\Phi = \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)S}{N(t)}$. The proposed model's flowchart and parameters description are well explained in (4).

Variable	Description
$S(t)$	Susceptible class
$E(t)$	Exposed class
$I(t)$	Infected
$Q(t)$	Quarantine class
$R(t)$	Recovered class
θ	Recruitment rate into susceptible population
μ	Natural mortality rate
δ	COVID-19 death rate
ω	Progression rate from exposed to infectious class
σ	Rate of loss of immunity
τ	Treatment rate for infectious individuals
ϕ	Treatment rate for quarantine individuals
ψ	Proportion of individuals that maintain social distancing
υ	Usage of a face mask and hand sanitizer by aportion of the population
ρ	Rate of recovery from infection
α_C	Effective transmission rate.

(4)

the $\alpha^{th}-$order Caputo fractional difference system associate to the system (3) is written as follows :

$$\begin{cases} {}^C\Delta_a^\alpha S(t) = & \theta - \frac{\alpha_c(1-\psi)(1-\upsilon)(E(t-1+\alpha)+I(t-1+\alpha))S(t-1+\alpha)}{N(t-1+\alpha)} - \mu S(t-1+\alpha) + \sigma R(t-1+\alpha), \\ {}^C\Delta_a^\alpha E(t) = & \frac{\alpha_c(1-\psi)(1-\upsilon)(E(t-1+\alpha)+I(t-1+\alpha))S(t-1+\alpha)}{N(t-1+\alpha)} - (\mu+\omega)E(t-1+\alpha), \\ {}^C\Delta_a^\alpha I(t) = & \omega E(t-1+\alpha) - (\mu+\delta+\rho+\tau)I(t-1+\alpha), \\ {}^C\Delta_a^\alpha Q(t) = & \rho I(t-1+\alpha) - (\mu+\delta+\phi)Q(t-1+\alpha), \\ {}^C\Delta_a^\alpha R(t) = & \phi Q(t-1+\alpha) + \tau I(t-1+\alpha) - (\sigma+\mu)R(t-1+\alpha), \end{cases} \quad t \in \mathbb{N}_{a+1-\alpha}, \tag{5}$$

where $0 < \alpha < 1$.

4. EXISTENCE AND UNIQUENESS

Now, to show the existence and uniqueness we use fixed point theory and Picard Lindelöf method. To proceed, we may rewrite the system described in (5) in the following classical form :

$$\begin{cases} {}^C\Delta_a^\alpha X(t) = F(t-1+\alpha, X(t-1+\alpha)), \\ X(a) = X_0, \end{cases} \tag{6}$$

where $t \in \mathbb{N}_{a+1-\alpha}^{T_{\max}}$,$((T_{\max} - a - 1 + \alpha) \in \mathbb{N})$ the vector $X(t) = (S(t), E(t), I(t), Q(t), R(t))^T$ and the function $F(t, X(t))$ is defined as follows :

$$\begin{array}{ll} F_1(t,S) = & \theta - \frac{\alpha_c(1-\psi)(1-\upsilon)(E(t)+I(t))S(t)}{N(t)} - \mu S(t) + \sigma R(t) \\ F_2(t,E) = & \frac{\alpha_c(1-\psi)(1-\upsilon)(E(t)+I(t))S(t)}{N(t)} - (\mu+\omega)E(t) \\ F_3(t,I) = & \omega E(t) - (\mu+\delta+\rho+\tau)I(t) \\ F_4(t,Q) = & \rho I(t) - (\mu+\delta+\phi)Q(t) \\ F_5(t,R) = & \phi Q(t) + \tau I(t) - (\sigma+\mu)R(t) \end{array} \tag{7}$$

To do so we proceed in the following manner. Using initial conditions $(X(a))$ and fractional sum operator (1), we transform the system (5) into the following sum equations :

$$\left\{ \begin{array}{ll} S(t) - S(a) = & \Delta_a^{-\alpha} \left(\theta - \frac{\alpha_c(1-\psi)(1-\upsilon)(E(t-1+\alpha)+I(t-1+\alpha))S(t-1+\alpha)}{N(t-1+\alpha)} - \mu S(t-1+\alpha) + \sigma R(t-1+\alpha) \right), \\ E(t) - E(a) = & \Delta_a^{-\alpha} \left(\frac{\alpha_c(1-\psi)(1-\upsilon)(E(t-1+\alpha)+I(t-1+\alpha))S(t-1+\alpha)}{N(t-1+\alpha)} - (\mu+\omega)E(t-1+\alpha) \right), \\ I(t) - I(a) = & \Delta_a^{-\alpha} \left(\omega E(t-1+\alpha) - (\mu+\delta+\rho+\tau)I(t-1+\alpha) \right), \\ Q(t) - Q(a) = & \Delta_a^{-\alpha} \left(\rho I(t-1+\alpha) - (\mu+\delta+\phi)Q(t-1+\alpha) \right), \\ R(t) - R(a) = & \Delta_a^{-\alpha} \left(\phi Q(t-1+\alpha) + \tau I(t-1+\alpha) - (\sigma+\mu)R(t-1+\alpha) \right), \end{array} \right. \tag{8}$$

for $t \in \mathbb{N}_{a+1-\alpha}^{T_{\max}}$. Using (7) and the definition of $\Delta_a^{-\alpha}$ in (8), we obtained the state variable in terms of $F_i(t, X(t))$, where $i = 1 \cdots 6$.

$$\left\{ \begin{array}{ll} S(t) = & S(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_1(s-1+\alpha, S(s-1+\alpha)), \\ E(t) = & E(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_2(s-1+\alpha, E(s-1+\alpha)), \\ I(t) = & I(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_3(s-1+\alpha, I(s-1+\alpha)), \qquad t \in \mathbb{N}_{a+1-\alpha}^{T_{\max}}. \\ Q(t) = & Q(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_4(s-1+\alpha, Q(s-1+\alpha)), \\ R(t) = & R(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_5(s-1+\alpha, R(s-1+\alpha)). \end{array} \right. \tag{9}$$

The Picard iterations are given by the following equations :

$$\left\{ \begin{array}{ll} S_{n+1}(t) = & S(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_1(s-1+\alpha, S_n(s-1+\alpha)) \\ E_{n+1}(t) = & E(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_2(s-1+\alpha, E_n(s-1+\alpha)) \\ I_{n+1}(t) = & I(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_3(s-1+\alpha, I_n(s-1+\alpha)) \qquad t \in \mathbb{N}_{a+1-\alpha}^{T_{\max}}. \\ Q_{n+1}(t) = & Q(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_4(s-1+\alpha, Q_n(s-1+\alpha)) \\ R_{n+1}(t) = & R(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F_5(s-1+\alpha, R_n(s-1+\alpha)) \end{array} \right. \tag{10}$$

Corresponding to the form (9), and with the initial condition we have the following sum equation :

$$X(t) = X(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F(s-1+\alpha, X(s-1+\alpha)). \; t \in \mathbb{N}_{a+1-\alpha}. \tag{11}$$

Lemma 1 *The function $F(t, X(t))$ defined in (7) satisfies the Lipschitz condition given by*

$$\|F(t, X(t) - F(t, X(t))\| \leq \beta \, \|(X(t) - X(t))\|, \tag{12}$$

where

$$\beta = \max\left\{ \begin{array}{c} \|\alpha_c(1-\psi)(1-\upsilon)+\mu\|, \|\alpha_c(1-\psi)(1-\upsilon)-(\mu+\omega)\|, \\ \|(\mu+\delta+\rho+\tau)\|, \|(\mu+\delta+\phi)\|, \|(\sigma+\mu)\| \end{array} \right\}. \tag{13}$$

Proof. Summarizing that $S_1(t)$ and $S_2(t)$ are couple functions, we reach

$$\|F_1(t,S_1)-F_1(t,S_2)\| = \left\| \left(\frac{\alpha_c(1-\psi)(1-\upsilon)(E(t)+I(t))}{N(t)} + \mu \right)(S_1(t)-S_2(t)) \right\|. \tag{14}$$

Taking into account

$$\beta_1 = \|\alpha_c(1-\psi)(1-\upsilon)+\mu\|, \tag{15}$$

one reaches

$$\|F_1(t,S_1)-F_1(t,S_2)\| \leq \beta_1 \|(S_1(t)-S_2(t))\|. \tag{16}$$

Continuing in the same way, one gets

$$\begin{array}{ll} \|F_2(t,E)-F_1(t,E^*)\| \leq & \beta_2 \|(E(t)-E^*(t))\|, \\ \|F_3(t,I)-F_1(t,I^*)\| \leq & \beta_3 \|(I(t)-I^*(t))\|, \\ \|F_4(t,Q)-F_1(t,Q^*)\| \leq & \beta_4 \|(Q(t)-Q^*(t))\|, \\ \|F_5(t,R)-F_1(t,R^*)\| \leq & \beta_5 \|(R(t)-R^*(t))\|, \end{array} \tag{17}$$

where

$$\begin{array}{ll} \beta_2 = & \|\alpha_c(1-\psi)(1-\upsilon)-(\mu+\omega)\|, \\ \beta_3 = & \|(\mu+\delta+\rho+\tau)\|, \\ \beta_4 = & \|(\mu+\delta+\phi)\|, \\ \beta_5 = & \|(\sigma+\mu)\|, \end{array} \tag{18}$$

From (16-17), we find that the kernels F_1, F_2, F_3, F_4 and F_5 is satisfying the Lipschitz condition, moreover if $\beta_i < 1$, for $i = 1,2,3,4,5$ then the kernel F_i for $i = 1,2,3,4,5$ is contraction. ∎

Theorem 2 *Assuming we have (13), then there exist a unique solution to the system (5) if*

$$\beta \left| (T_{\max}-a)^{(\alpha)} - (1-\alpha)^{(\alpha)} \right| < 1. \tag{19}$$

Proof. The solution to the system (6) is

$$X(t) = P(X(t)), \tag{20}$$

where, P is the Picard operator defined by

$$P(X(t)) = X(a) + \frac{1}{\Gamma(\alpha)} \sum_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} F(s-1+\alpha, X(s-1+\alpha)). \tag{21}$$

Further, we have

$$\begin{array}{rcl} \|P(X_1(t)-P(X_2(t))\| & = & \left\| \frac{1}{\Gamma(\alpha)} \sum\limits_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} (F(s-1+\alpha, X_1(s-1+\alpha)) \right. \\ & & \left. -F(s-1+\alpha, X_2(s-1+\alpha)))\right\|, \\ & \leq & \frac{1}{\Gamma(\alpha)} \sum\limits_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} \|(F(s-1+\alpha, X_1(s-1+\alpha)) \\ & & -F(s-1+\alpha, X_2(s-1+\alpha)))\|, \\ & \leq & \frac{1}{\Gamma(\alpha)} \left(\sum\limits_{s=a}^{t-\alpha} (t-s-1)^{(\alpha-1)} \right) \max\limits_{s\in\mathbb{N}_a^{t-\alpha}} \|(F(s-1+\alpha, X_1(s-1+\alpha)) \\ & & -F(s-1+\alpha, X_2(s-1+\alpha)))\|, \\ & \leq & \frac{(t-a)^{(\alpha)}-(1-\alpha)^{(\alpha)}}{\Gamma(\alpha)} \beta \|(X_1(t)-X_2(t))\|. \end{array} \tag{22}$$

Since, $\frac{(T_{\max}-a)^{(\alpha)}-(1-\alpha)^{(\alpha)}}{\Gamma(\alpha)}\beta < 1$, $(t \leq T_{\max})$ then, the operator P is a contraction, hence the system (6) has a unique solution. ■

For the non negative solution we need the following lemma

Lemma 3 *Let $\alpha > 0$, $\alpha \notin \mathbb{N}$ and f is defined on $\mathbb{N}_a$. Then :*

$$f(t) = f(a) + \frac{1}{\Gamma(\alpha)} \sum_{r=a+1-\alpha}^{t-\alpha} (t-r-1)^{(\alpha-1)} \ {}^{C}\Delta_a^{\alpha} \ f(r), \quad \forall t \in \mathbb{N}_{a+1}, \tag{23}$$

Remark 1 *From Lemma 3 we have*

1. If ${}^{C}\Delta_a^{\alpha} f(t) \geq 0$ then f is non decreasing for all $t \in \mathbb{N}_a$.

2. If ${}^{C}\Delta_a^{\alpha} f(t) \leq 0$ then f is non increasing for all $t \in \mathbb{N}_a$.

Theorem 4 *the solution of (5) is positive*

Proof. to proof that solutions of the system (5) with non negative initial data will remain non negative for all $t \in \mathbb{N}_a$, we use lemma ... . Since :

$$\begin{array}{ll} {}^{C}\Delta_a^{\alpha} S\big|_{S=0} = & \theta + \sigma R \geq 0, \\ {}^{C}\Delta_a^{\alpha} E\big|_{E=0} = & \frac{\alpha_c(1-\psi)(1-\upsilon)IS}{N} \geq 0, \\ {}^{C}\Delta_a^{\alpha} I\big|_{I=0} = & \omega E \geq 0, \\ {}^{C}\Delta_a^{\alpha} Q\big|_{Q=0} = & \rho I \geq 0, \\ {}^{C}\Delta_a^{\alpha} R\big|_{R=0} = & \phi Q + \tau I \geq 0. \end{array} \tag{24}$$

Then we conclude that the solution $X(t) = (S(t), E(t), I(t), Q(t), R(t))^T$ of system (5) belongs to $\mathbb{R}_+^5$. ■

5. STABILITY ANALYSIS OF DISEASE FREE EQUILIBRIUM (DFE)

To evaluate the equilibrium let
${}^{C}\Delta_a^{\alpha} S(t) = {}^{C}\Delta_a^{\alpha} E(t) = {}^{C}\Delta_a^{\alpha} I(t) = {}^{C}\Delta_a^{\alpha} Q(t) = {}^{C}\Delta_a^{\alpha} R(t) = 0$. System (5) become :

$$\left\{ \begin{array}{c} \theta - \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)S}{N} - \mu S + \sigma R = 0 \\ \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)S}{N} - (\mu+\omega)E = 0 \\ \omega E - (\mu+\delta+\rho+\tau)I = 0 \\ \rho I - (\mu+\delta+\phi)Q = 0 \\ \phi Q + \tau I - (\sigma+\mu)R = 0 \end{array} \right. \tag{25}$$

the jacobian matrix is :

$$J = \begin{pmatrix} -\frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)}{N} - \mu & -\frac{\alpha_c(1-\psi)(1-\upsilon)S}{N} & -\frac{\alpha_c(1-\psi)(1-\upsilon)S}{N} & 0 & \sigma \\ \frac{\alpha_c(1-\psi)(1-\upsilon)(E+I)}{N} & \frac{\alpha_c(1-\psi)(1-\upsilon)S}{N} - (\mu+\omega) & \frac{\alpha_c(1-\psi)(1-\upsilon)S}{N} & 0 & 0 \\ 0 & \omega & -(\mu+\delta+\rho+\tau) & 0 & 0 \\ 0 & 0 & \rho I & -(\mu+\delta+\phi) & 0 \\ 0 & 0 & \tau & \phi & -(\sigma+\mu) \end{pmatrix} \tag{26}$$

The following results provide the local and global stability results of the system (5) around the DFE. For which we get the DFE as follows :

$$e_f = (S_0, 0, 0, 0, 0) = (\frac{\theta}{\mu}, 0, 0, 0, 0). \tag{27}$$

5.0.1. Local stability

Theorem 5 *[3] Let $\alpha \in (0,1)$ and A is an $n \times n$ constant matrix. If $\lambda \in S^\alpha$ for all the eigenvalues λ of A, then the trivial solution of ([**?**]) is asymptotically stable. In this case, the solutions of ([**?**]) decay towards zero algebraically (and not exponentially), more precisely*

$$\|x(t)\| = O(t^{-\alpha}) \quad as\ t \to \infty,$$

*for any solution x of ([**?**]).*

*Furthermore, if $\lambda \in \mathbb{C} \backslash\ cl(S^\alpha)$ for an eigenvalue λ of A, the zero solution of ([**?**]) is not stable.*

The respective Jacobian of above matrices at the DFE are evaluated as follows :

$$J_{e_f} \quad = \begin{pmatrix} -\mu & -b_1 & -b_1 & 0 & \sigma \\ 0 & b_1 - b_2 & b_1 & 0 & 0 \\ 0 & \omega & -b_3 & 0 & 0 \\ 0 & 0 & 0 & -b_4 & 0 \\ 0 & 0 & \tau & \phi & -b_5 \end{pmatrix}, \tag{28}$$

where $b_1 = \alpha_c(1-\psi)(1-\upsilon), b_2 = (\mu+\omega), b_3 = (\mu+\delta+\rho+\tau), b_4 = (\mu+\delta+\phi), b_5 = (\sigma+\mu)$.

Theorem 6 *The DFE of the system (5) is locally asymptotically stable when :*

$$\mu + \delta + \sigma + \phi < 2^\alpha, \tag{29}$$

$$b_1(\frac{b_3+\omega}{b_3}) < b_2, \tag{30}$$

$$b_1 < b_2 + b_3, \tag{31}$$

$$b_2 + b_3 - b_1 < 2^\alpha, \tag{32}$$

Proof. Computations give the following characteristic polynomial :

$$-(\lambda+\mu)(\lambda+b_4)(\lambda+b_5)\left(\lambda^2 + (b_2 - b_1 + b_3)\lambda + (b_2 b_3 - b_1 b_3 - \omega b_1)\right), \tag{33}$$

In the characteristic polynomial of J_{e_f} we have the eigenvalues :

$$\begin{aligned} \lambda_1 = &\quad -\mu, \\ \lambda_2 = &\quad -b_4, \\ \lambda_3 = &\quad -b_5, \\ \lambda_4 = &\quad \tfrac{1}{2}(b_1 - b_2 - b_3 - \xi), \\ \lambda_5 = &\quad \tfrac{1}{2}(b_1 - b_2 - b_3 + \xi), \end{aligned} \tag{34}$$

where $\xi = \sqrt{(b_2 - b_1 - b_3)^2 + 4\omega b_1}$. According to assumptions (29) :

$$\begin{array}{c} -2^\alpha < \lambda_1 < 0, \\ -2^\alpha < \lambda_2 < 0, \\ -2^\alpha < \lambda_3 < 0. \end{array} \tag{35}$$

According to assumptions (30) :

$$\begin{array}{c} b_1(\frac{b_3+\omega}{b_3}) < b_2, \\ 4b_1b_3 + 4\omega b_1 < 4b_2b_3, \\ 2b_1b_3 - 2b_2b_3 - 2b_1b_2 + 4\omega b_1 < 2b_2b_3 - 2b_1b_3 - 2b_1b_2, \\ (b_2 - b_1 - b_3)^2 + 4\omega b_1 < (b_2 + b_3 - b_1)^2, \\ \xi < b_2 + b_3 - b_1, \end{array} \tag{36}$$

$\Rightarrow$

$$\lambda_4 < \lambda_5 < 0. \tag{37}$$

According to assumptions (31)-(32) :

$$-2^\alpha < (b_1 - b_2 - b_3) < \lambda_4 < \lambda_5, \tag{38}$$

by theorem ... DFE of the system (5) is locally asymptotically stable. ■

5.0.2. *Global stability*

Theorem 7 *[4] If there exists a positive definite and decrescent scalar function $V(t,x)$ such that*

$${}^C\Delta_{t_0}^\alpha V(t, x(t)) \leq 0, \tag{2.34}$$

*for all $t_0 \in \mathbb{N}_a$, then the trivial solution of ([**?**]) is uniformly stable.*

Theorem 8 *If*

$$\frac{(b_2 - b_1)\,b_3}{b_1\omega} > 1 \tag{39}$$

then, in the absence of COVID-19, that is, the COVID-19 free equilibrium $(S_0, 0, 0, 0, 0)$ is globally asymptotically stable.

Proof. Lyapunov function is commonly used to proof the Global Stability of the Disease Free Equilibrium [5]. Consider the formed Lyapunov function of the type

$$L(t) = AE + I, \tag{40}$$

withe

$$\frac{\omega}{b_2 - b_1} < A < \frac{b_3}{b_1}. \tag{41}$$

Let's differentiate L with respect to time to have

$$\begin{aligned} {}^C\Delta_a^\alpha L(t) \quad &= A\,{}^C\Delta_a^\alpha E(t) + {}^C\Delta_a^\alpha I(t), \\ &= A\left(b_1 \tfrac{S}{N}(E + I) - b_2 E\right) + (\omega E - b_3 I), \\ &= \left(\omega - Ab_2 + A\tfrac{S}{N}b_1\right)E + \left(-b_3 + A\tfrac{S}{N}b_1\right)I, \\ &\leq (\omega + Ab_1 - Ab_2)E + (Ab_1 - b_3)I, \\ &\leq 0. \end{aligned} \tag{42}$$

■

6. CONCLUSIONS

In this chapter, a discrete fractional order covid model is studied and some results of local and global stability are given by using an appropriate Lyapunov function to ensure the disappearance of the disease in comfortable conditions.

7. REFERENCES

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