CONTROL OF AN EULER-BERNOULLI BEAM WITH A NONLINEAR TENSION AND AN END-MASS

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ABSTRACT

In this article, we intend to study the vibrations of a nonlinear Euler-Bernoulli beam without internal damping, fixed at one end and a mass attached at the other end. By applying a suitable control at the free end, we can quickly dampen these vibrations. In fact, we give the result of the exponential stability of the solutions, and we base our method on the multiplier technique.

Keywords : exponential stability, Euler-Bernoulli beam, boundary control, nonlinear tension, multiplier method.

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1. INTRODUCTION

In this paper, we consider a Euler-Bernoulli beam with a nonlinear tension and an mass attached to its free end, The beam is modeled through the following equation :

$$\begin{cases} \rho w_{tt} + EIw_{xxxx} - Tw_{xx} - \frac{3EA}{2} w_x^2 w_{xx} = 0, & x \in (0,L), t \geq 0, \\ w(0,t) = w_x(0,t) = w_{xx}(L,t) = 0, & \\ Mw_{tt}(L,t) - EIw_{xxx}(L,t) + Tw_x(L,t) + \frac{EA}{2} w_x^3(L,t) = U(t) & t \geq 0, \\ w(x,0) = w^0(x), \qquad w_t(x,0) = w^1(x), & x \in (0,L), \end{cases} \tag{1}$$

where $w(x,t)$ represents the transverse displacements of the beam at a position x for time t, $\rho > 0$ is the uniform mass per unit length of the beam, and L is the length of the beam. Also, T is the axial tension of the beam, EI is the bending stiffness, EA is the axial stiffness, and the control force $U(t)$ will be specified later.

Recent years have witnessed an increasing cognizance of the problem of controlling thinner structures in various industries, especially slender ones. Domains like transportation and construction have particularly benefited from it. The purpose behind attempts to control these structures is to suppress or at least reduce transverse vibrations which often occur due to the irregular material property or environmental disturbances.

The issue of stability and stabilization has been discussed by many researchers who relied in their studies on several damping terms that induce dissipation. For works that relied on internal damping in order to achieve stability of the system, [1, 7, 9, 10, 11, 12, 17, 18] can offer decent insights.

As for the boundary stabilization case, we have [3, 4, 8, 14, 16]. Furthermore, for studies that tackle the stabilization and control of Euler-Bernoulli beams with damping, we may mention the works of Gao et al. [2], Karagiannis et al. [6] and Miletić et al. [13].

We note also that in [5] He et al. adopted the linear system of (1) with internal viscous damping term cw_t. Thus, they established exponential stability under a robust control.

Seghour et al. [15] investigated the linear system of (1) with the viscoelastic damping term $\int_0^t h(t-s)w_{xxxx}(x,s)ds$. The result was establishing an exponential stability result under a nonlinear control.

The main contribution of this work can be summarized in achieving an exponential stability result result strictly with linear boundary control and without internal damping for the nonlinear problem.

2. PRELIMINARY

In this paper, $\|.\|$ represents the norm and $(.;.)$ is the inner product of $L^2(0,L)$. We introduce the energy associated to (1) by

$$E(t) = \frac{\rho}{2}\|w_t\|^2 + \frac{M}{2}w_t(L,t) + \frac{EI}{2}\|w_{xx}\|^2 + \frac{T}{2}\|w_x\|^2 + \frac{EA}{8}\|w_x^2\|^2. \tag{2}$$

Observe that this is the usual classical energy. The first two terms represent kinetic energy while the rest of the terms represent potential energy.

Control

The control objective is to reduce the free transverse vibrations of the beam. Lyapunov's direct method is used to construct a suitable boundary control at the free boundary of the flexible beam.

To stabilize system (1)), we propose the following control :

$$U(t) = -k_1 w_t(L,t) - k_2 w_{xt}(L,t), \tag{3}$$

where k_1 and k_2 are positive constants.

Lemma 1 *The energy functional* (2) *satisfies*

$$E'(t) = w_t(L,t)U(t), \qquad \forall t \geq 0. \tag{4}$$

Proof. Taking the inner product of the first equation of (1) with w_t in $L^2(0,L)$, integrating by parts and taking into account the boundary, we get (4). ■

3. STABILITY RESULT

In order to prove the energy decay result. let us define the Lyapunov functional by

$$\mathscr{L}(t) = E(t) + \beta V(t), \tag{5}$$

where β is a positive constant and

$$V(t) = (xw_x, w_t) + LMw_t w_x(L,t) + Lk_2 w_x^2(L,t). \tag{6}$$

Proposition 2 *There exist two positive constants a and b, such that*

$$aE(t) \leq \mathscr{L}(t) \leq bE(t), \qquad \forall t \geq 0, \tag{7}$$

i.e. $E(t) \sim \mathscr{L}(t)$.

Proof. This Proposition can be proved by applying Young, Cauchy-Schwarz and Poincaré's inequalities, as follows

$$\begin{aligned} |V(t)| &\leq \frac{L}{2}\|w_t\|^2+\frac{L}{2}\|w_x\|^2+\frac{LM}{2}w_t^2(L,t)+\left(\frac{LM}{2}+Lk_2\right)w_x^2(L,t) \\ &\leq \frac{L}{2}\|w_t\|^2+\frac{L}{2}\|w_x\|^2+\frac{LM}{2}w_t^2(L,t)+\left(\frac{LM}{2}+Lk_2\right)L\|w_{xx}\|^2 \\ & \theta E(t), \end{aligned} \tag{8}$$

where $\theta=\max\{\frac{L}{\rho},\frac{L}{T},L,\frac{LM+2Lk_2}{EI}\}$.

Considering $\beta<1/\theta$ and using a proper selections, we can get (7) with $a=1-\beta\theta$ and $b=1+\beta\theta$. ∎

Lemma 3 *The derivative of $V_1(t)$ satisfies along solutions of system* (1)

$$\begin{aligned} V'(t) &\leq -\frac{1}{2}\|w_t\|^2-\frac{3EI}{2}\|w_{xx}\|^2-\frac{T}{2}\|w_x\|^2-\frac{3EA}{8}\|w_x^2\|^2+MLw_tw_{tx}(L,t) \\ &+\left[\frac{L}{2}+\frac{Lk_1}{\delta}\right]w_t^2(L,t)-\left[\frac{TL}{2}-\frac{Lk_1}{4}\delta\right]w_x^2(L,t)-\frac{EAL}{8}w_x^4(L,t) \end{aligned} \tag{9}$$

Proof. By differentiating V_1, taking into account system (1) and integrating by parts, we get

$$\begin{aligned} V'(t) &= \frac{L}{2}w_t^2(L,t)-\frac{1}{2}\|w_t\|^2-\frac{3EI}{2}\|w_{xx}\|^2-\frac{T}{2}\|w_x\|^2-\frac{3EA}{8}\|w_x^2\|^2 \\ &-\frac{TL}{2}w_x^2(L,t)-\frac{EAL}{8}w_x^4(L,t)+MLw_tw_{tx}(L,t)-Lk_1w_tw_x(L,t). \end{aligned} \tag{10}$$

After applying the inequality of young to the last term of the above equation, for $\delta>0$, we obtain (9). ∎

Theorem 4 *The energy $E(t)$ satisfies along the solution of system* (1)

$$E(t)\leq Ae^{-\lambda t}, \qquad t\geq 0, \tag{11}$$

where A and λ are two positive constants.

Proof. Taking the derivative of $\mathscr{L}(t)$, (3), (4) and (9), and picking $k_2=ML$, we obtain

$$\mathscr{L}'(t)\leq -cE(t). \tag{12}$$

Using equivalence relation (7), the above inequality becomes

$$\mathscr{L}'(t)\leq -\frac{c}{b}\mathscr{L}(t). \tag{13}$$

which gives

$$\mathscr{L}(t)\leq \mathscr{L}(0)e^{-\frac{c}{b}t}. \tag{14}$$

By reusing relation (7), we find (11), in which $A=\frac{\mathscr{L}(0)}{b}$ and $\lambda=\frac{c}{b}$. ∎

4. CONCLUSIONS

This research succeeded in obtaining a linear boundary control in order to achieve the exponential stability of the nonlinear Euler-Bernoulli beam under free vibrations.

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