DECOMPOSITION BASED PARALLEL HYBRID MOEA WITH APPLICATION TO THE MULTIOBJECTIVE MULTIDIMENSIONAL KNAPSACK PROBLEM

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ABSTRACT

Recently, there has been a noticeable tendency in research for combinatorial optimization issues toward the hybridization of metaheuristics with other optimization techniques. On the other hand, parallel conception of multiobjective evolutionary algorithms (MOEAs) provides a significant enhancements in terms of efficiency and effectiveness. In this paper, we propose a hybrid parallel multiobjective evolutionary algorithm, with an application to the multiobjective multidimensional Knapsack Problem (MOMKP). The suggested approach can be considered as an enhanced parallel variant of two-phase method. Finally, we present an experimental study, where we assess the suggested approach against state-of-the-art sequential and parallel MOEAs, as to emphasize the contribution of the search strategy of the parallel MOEAs and its ability to approximate target areas of the true Pareto Front.

1. INTRODUCTION

Multiobjective Problems consists to optimize k objective functions simultaneously. The general form of MOPs is stated as follows :

$$\begin{cases} \text{"}max\text{"} Z(x) = (Z^1(x), Z^2(x), \dots, Z^k(x)), \\ s.t., \quad x \in \Omega. \end{cases}$$

where Ω is the decision space, $x \in \Omega$ is a decision vector, and the vector $Z(x)$ consists of k objective functions $Z^i(x) : \Omega \rightarrow \mathbb{D}_i$, $i \in \{1,...,k\}$. Since the aim in MOPs is to find good compromises. Here, we present the dominance relation, as to define optimality in MOPs. For any couple of feasible solutions x and x' in Ω, the vector $Z(x) = (Z^1(x),...,Z^k(x))$ is said to dominate the vector $Z(x^{'}) = (Z^1(x^{'}),\dots,Z^k(x^{'}))$, denoted as $Z(x) \succ Z(x^{'})$, if and only if, $\forall i \in \{1,\dots,k\}$, $Z^i(x) \leq Z^i(x^{'})$ and $Z(x) \neq Z(x^{'})$. A feasible solution $x^* \in \Omega$ is called a Pareto optimal solution or an efficient solution, if and only if, $\nexists y \in \Omega$ such that $Z(y) \succ Z(x^*)$. The set of Pareto optimal solutions is called the Pareto-optimal Set (PS) : $PS = \{x \in \Omega | \nexists y \in \Omega, Z(y) \succ Z(x^*)\}$. The evaluation of solutions in PS is called the Pareto Front (PF) : $PF = \{Z(x) | x \in PS\}$.

Furthermore, there exists an important classification of efficient solutions : supported efficient solutions and non-supported efficient solutions. According to Geoffrion's theorem [8], the supported efficient solutions, denoted X_{SE}, can be obtained by solving the parametric single-objective problems obtained by a linear aggregation of the different objectives P_λ :

$$(P_\lambda) \begin{cases} max \sum_{i=1}^{k} \lambda_i Z^i(x), \\ s.t., \quad x \in \Omega, \end{cases}$$

where, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k) \in \mathbb{R}^k_+$ is a weight vector with all positive components.

On the other hand, there exists the non-supported efficient solutions set, denoted X_{NS}, this subset of the efficient solutions set cannot be obtained by solving P_λ. Furthermore, the images of the unsupported solutions are not located on the boundary of the convex envelope.

In this paper, we propose a parallel hybrid multiobjective evolutionary algorithm, designed in a master/salve model, we call it Decomposition Based Parallel Hybrid MOEA (D/PHMOEA). The suggested algorithm is an enhanced two-phase type algorithm, where the first phase consists of finding the supported solutions set using an exact method. In the second phase, the decision space is structurally decomposed and allocated to multiple MOEAs operating in parallel. Each MOEA is dedicated to a specific region of the decision space that is initially characterized by a subset of the supported solutions found in the first phase. We assess the suggested method against some successful MOEAs using benchmark instances of the Multiobjective Multidimensional Knapsack Problem (MOMKP). This latter is a variant of the Knapsack Problem (KP), which is known to be NP-hard [6]. Mathematically, MOMKP can be stated as follows : given n items having p characteristics (weight, volume, etc.) $w^i_j \geq 0$, where, $j \in \{1,\dots,p\}$, $i \in \{1,\dots,n\}$, and m profits $c^k_j,\ k \in \{1,\dots,m\}$, we want to select items as to maximize the m total profits, while not exceeding the p knapsack capacities W_i with regards to the different characteristics. The MOMKP is formulated as follows :

$$(MOMKP)\begin{cases} \text{"}max\text{"}\ Z^k(x) = \sum_{j=1}^{n} c^k_j x_j, \quad k \in \{1,\dots,m\} \\ s.t., \quad \sum_{j=1}^{n} w^i_j x_j \leq W_i, \quad j \in \{1,\dots,p\} \\ x_j \in \{0,1\}, \forall j \in \{1,\dots,n\}. \end{cases}$$

2. SUGGESTED ALGORITHM (D/PHMOEA)

2.1. Description

In this section, we present a resumed description of the suggested algorithm, which is, as we already mentioned, an enhanced variant of the two-phase method. The first phase of the suggested algorithm method remains unchanged, as it is the case for all two-phase algorithms. It consists in the construction of the set of efficient solutions supported by the dichotomy method proposed by Aneja & Nair [9], based on Geoffrion's theorem [8]. This algorithm generates all the supported efficient solutions, including extreme and non-extreme ones, using a single objective problem whose objective function is a linear aggregation of two objectives (see P_λ in the introduction). Next, after having the set of supported efficient solutions in hand, the second phase consists of approximating the set of non-supported solutions using multiple asynchronous parallel MOEAs. Each one of the parallel search entity is designed to target a specific region of the Pareto optimal front. This is by initializing its archive solutions set using a subset of the supported efficient solutions set gathered form the same region. Furthermore, the selection operator is defined according to the following order relation : let P_t be the current population of a search entity, PS_t be the set of Pareto solutions obtained at iteration t (i.e., $= \{x \in P_t |\ \nexists y \in P_t : y \succ x\}$), and $R \subset Z(PS_t) \cap Z(X_{SE})$ the extreme points enclosing the predefined region for the search entity, $|R| = k$ the number of objective functions. The order relation is defined as follows :

$$\forall x, y \in P_t, x \geq y \iff (x \succeq y) \vee (\phi(x) \geq \phi(y)),$$

where,

$$\phi(x) = \sum_{i=1}^{k} \left(Z^i(x) \sum_{j=1}^{k} \frac{R^i_j}{||\sum_{j=1}^{k} R_j||_2} \right).$$

Hence, the process of selecting individuals that pass to the next generation P_{t+1} is given explicitly as follows :

$$P_{t+1} = \{x \in P_t | (x \in PS_t) \vee (rank(x, P_t \backslash PS_t \leq N))\},$$

where, $rank(x,P)$ is the order of a solution x compared to elements of a set P according to the function ϕ, and N is the parameter fixing the size of the current directing population.

The suggested pMOEA can be classified as a cooperative algorithmic level parallel model designed in a master/worker paradigm, handling : (1) a master entity in charge of gathering and computing the global approximated Pareto solutions, (2) multiple MOEAs with directed the search to specific regions of the true Pareto front, with the help of a specific selection operator described above defined with a subset of supported efficient solutions. Regarding the decomposition procedure, this occurs over the decision space using the supported efficient solutions set found in the first phase. This is by partitioning this set into p equally sized sets, according to one of the objective functions. As we mentioned earlier, the extreme solutions of each subset is used to construct the selection operator of each parallel MOEAs.

Figure 1 presents an example of the decomposition procedure applied to a bi-objective Knapsack instance : 2KP100-TA-0 [10]. The decision space is decomposed into $p = 4$ sub-regions.

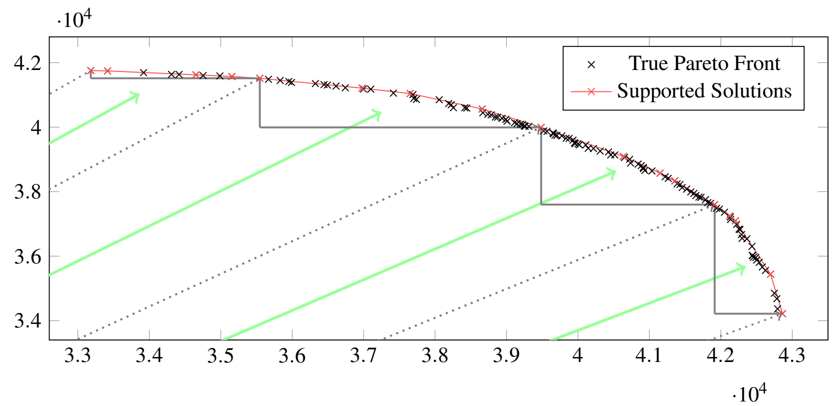

FIGURE 1 – Illustrative example of the used decision space decomposition (Bazgan KP instance [10], 2KP100-TA-0)

2.2. Experimental results

We tested the sugessted algorithm on benchmark instances of MOMKP chosen from the instance libraries : Zitzler and al. [7], of which we consider for this experiments three instances with the number of items 250, 500, and 750, with two objective functions. We compared the performance of the suggested algorithm three four multiobjective algorithms with different concepts and/or different search strategies : NSGAII [2], SPEA2 [3], MOEA/D [4], MOFPA [12], PCP-MOEA [11]. The evaluation and comparison of the obtained solution's the quality, one must consider (convergence, and the spread), we used three performance metrics : Inverted Generational Distance (IGD) [13], Hypervolume [7], and the set coverage metric [13].

Table 1 resumes the obtained values of the IGD metric assessing the convergence of the obtained Pareto sets. The IGD values shows clearly that, in general, D/PHMOEA converges better than all of the competing algorithms, especially for large instances, with the exception of the instance 2.250.

Instance	Algorithm				
	SPEA2	MOEA/D	MOFPA	PCPMOEA	D/PHMOEA
250	14.883	3.9690	0.7248	**0.2964**	0.5248
500	79.743	14.466	2.2850	0.7961	**0.3226**
750	224.794	32.655	10.062	3.1302	**2.1359**

TABLE 1 – Experimental results concerning the IGD metric of the MOMKP instances.

Table 2 resumes the obtained results regarding the Hypervolume indicator. This indicator is used to evaluate the convergence to the true Pareto front and diversity of the obtained Pareto front. As it is shown below, it is obvious that the suggested algorithm produces higher quality fronts, with significant difference especially when compared to SPEA2 and MOEA/D, and it's at least comparable to MOFPA and PCPMOEA algorithms.

Instance	Algorithm				
	SPEA2	MOEA/D	MOFPA	PCPMOEA	D/PHMOEA
250	9.1677527E+7	9.8374725E+7	9.8556257E+7	9.8654313e+7	**9.8692999E+7**
500	3.6944050E+8	4.0515241E+8	4.0707772E+8	4.0772607E+8	**4.0787113E+8**
750	7.8038570E+8	8.8553814E+8	8.8572075E+8	8.9260224E+8	**8.9351766e+8**

TABLE 2 – Experimental results concerning the Hypervolume indicator of the MOMKP instances.

Table 3 shows the obtained mean coverage values for each pair adduced as follows : the symboles $\succeq$ and $\preceq$ refer to $C(own\ algo.,\ competing\ algo.)$ and $C(competing\ algo.,\ own\ algo.)$ respectively. The results show that D/PHMOEA produces a better quality of Pareto fronts when compared to SPEA2, MOEA/D. However, PCPMOEA and MOFPA are shown to be the most competitive, especially for the smaller instances, although, the suggested algorithm maintained to be dominant, scoring an overall mean coverage values of 78% as dominant and 16% as dominated.

Instance		Algorithm			
		SPEA2	MOEA/D	MOFPA	PCPMOEA
250	$\succeq$	1	0.9930	0.6664	0.5783
	$\preceq$	0	0	0.2123	0.3772
500	$\succeq$	1	0.9995	0.8299	0.9177
	$\preceq$	0	0	0.0981	0.0047
750	$\succeq$	1	0.9985	0.8697	0.8531
	$\preceq$	0	0	0.0885	0.1122
Average	$\succeq$	1	0.9970	0.7886	0.7830
	$\preceq$	0	0	0.3989	0.1647

The last two rows contain the mean values for each column.

TABLE 3 – Coverage metric of the suggested algorithm against other competing algorithms.

Figure 2 presents an illustrative example of the obtained results. The visual observation confirms the fact that the suggested algorithm is at least comparable to recent state-of-the-art algorithms.

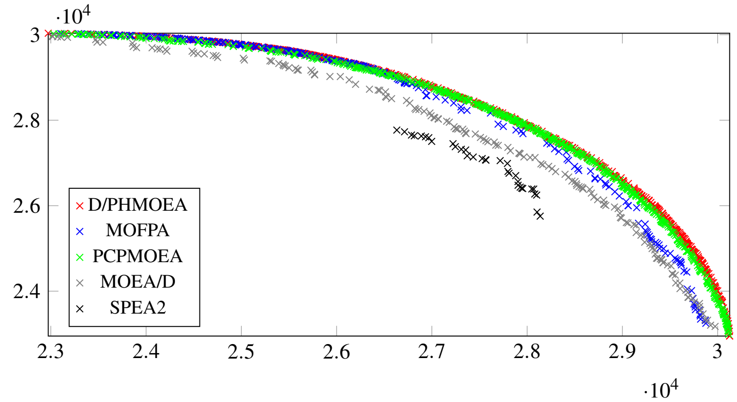

FIGURE 2 – Illustrative example of the obtained approximated Pareto fronts using SPEA2, MOEA/D, MOFPA, PCPMOEA, and D/PHMOEA.

3. CONCLUSIONS

In this paper, we presented a parallel two-phase type algorithm with an application to the multiobjective multidimensional Kanapsack Problem, called Decomposition based Parallel Hybrid MOEA (D/PHMOEA). The suggested algorithm is a hybrid algorithm, combining an exact method for finding the set of supported solutions, and a parallel MOEA with weighted-criteria selection operator, designed in a master/worker paradigm, as to target specific regions of the true Pareto set. The suggested algorithm has been assessed against state-of-the-art algorithms with different search strategies. The approach has shown conclusive results regarding the convergence and diversity of the evolved solutions.

4. REFERENCES

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